General Four-Resonator Filters at Microwave Frequencies

General four-resonator filters are capable of providing both band-pass and band-reject behavior. The potential advantages of general three and four-resonator filters have been discussed by Johnson, who considers *dissipationless* filters using inductive couplings.[1] Johnson has presented experimental data on a lumped-circuit-element general filter at 20 Mc/s, and has suggested some techniques for microwave implementation of general filters. Kurzrok has analyzed *dissipative* general three-resonator filters, and applied this theory to the development of a general three-resonator filter in X-band rectangular waveguide.[2]

General three-resonator filters can provide band-reject behavior on only one filter skirt. This asymmetrical selectivity is desirable for some applications. To achieve band-reject behavior on both filter skirts, a general four-resonator filter can be employed. In this correspondence, the performance capabilities of a microwave general four-resonator filter will be discussed.

The schematic of a lumped-circuit general four-resonator filter is shown in Fig. 1. To achieve band-reject behavior on both skirts, an additional bridging coupling, K_{14}, is required. The sense of the bridging coupling, K_{14}, must be opposite to the sense of all normal couplings (i.e., K_{12} and K_{23}). Failure to provide this phase reversal in the bridging coupling results in a degradation of selectivity on both filter skirts and no band-reject behavior.[1]

To realize a general filter at microwave frequencies, a four-resonator coaxial structure was selected. (See Fig. 2.) Quarter-wave TEM resonators were employed, using 1.375-inch square outer conductors and compound cylindrical center conductors. The center conductors consisted of 0.218-inch diameter slugs that can be moved longitudinally through 0.375-inch diameter fixed supports equipped with contacting fingers. Input/output coupling probes were fitted to type N connectors. Normal interstage couplings K_{12} and K_{23} were circular apertures located for minimum frequency sensitivity.[3] With $K_{14}=0$, ordinary band-pass behavior can be obtained. This type of filter structure is commonly used in commercially available filters designed for Butterworth or Chebyshev response shapes. Using 0.875-inch diameter apertures for K_{12} and a 0.719-inch diameter aperture for K_{23}, a monotonic response shape (not quite Butterworth) was obtained with a relative 3-dB bandwidth of 29 Mc/s. A 0.218 in diameter aperture connecting the first and fourth resonators resulted in negligible bridging coupling, K_{14}.

Manuscript received February 16, 1966.

[1] E. C. Johnson, "New developments in designing bandpass filters," *Electronic Industries*, pp. 87–90, 92, and 94, January 1964.

[2] R. M. Kurzrok, "General three-resonator filters in waveguide," *IEEE Trans. on Microwave Theory and Techniques (Correspondence)*, vol. MTT-14, pp. 46–47, January 1966.

[3] R. M. Kurzrok, "Design of interstage coupling apertures for narrow-band tunable coaxial band-pass filters," *IRE Trans. on Microwave Theory and Techniques (Correspondence)*, vol. MTT-10, pp. 143–144, March 1962.

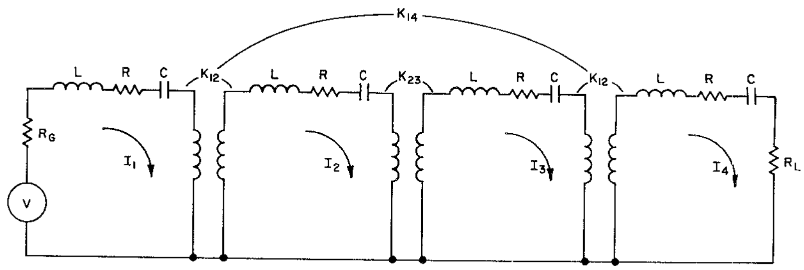

Fig. 1. Lumped-circuit schematic.

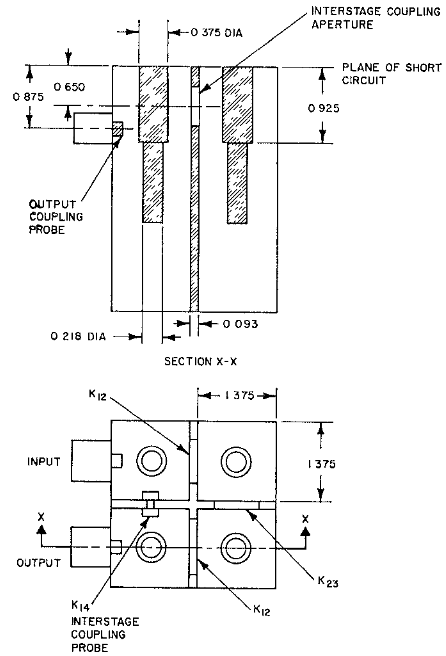

Fig. 2. Coaxial filter structure.

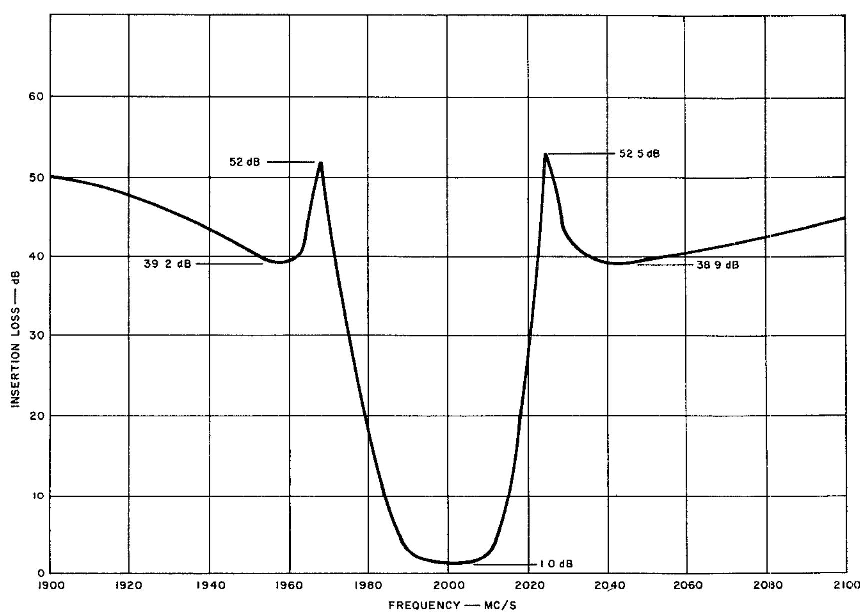

Fig. 3. General four-resonator filter response.

Upon adding a capacitive interstage coupling mechanism, appreciable bridging coupling, K_{14}, was realized, and the general filter response curve of Fig. 3 was obtained. Pass band insertion loss (primarily dissipation) was 1.0 dB and frequencies of peak rejection were realized on both filter skirts. It should be emphasized that realization of this performance depends upon using magnetic couplings (K_{12} and K_{23}) and electric coupling (K_{14}) that are in phase opposition. This type of coupling technique will not work for interdigital filter structures in which magnetic and electric couplings are in phase.

Measured data on the various filter couplings is tabulated below for different center frequencies f_0:[4]

Coupling	f_0=1900 Mc/s	f_0=2000 Mc/s	f_0=2100 Mc/s
Δf_{12}	24.8 Mc/s	25.0 Mc/s	26.0 Mc/s
Δf_{23}	14.4 Mc/s	15.5 Mc/s	16.1 Mc/s
Δf_{14}	2.8 Mc/s	3.3 Mc/s	4.0 Mc/s

The absolute coefficient of coupling between the ith and jth resonators will be $K_{ij} \cong \Delta f_{ij}/f_0$, where f_0 is the center frequency and Δf_{ij} is a coupling bandwidth. It can be seen that Δf_{12} and Δf_{23} are relatively insensitive to changes in center frequency, while Δf_{14} is quite frequency sensitive.

The normalized frequencies of peak rejection can be determined from (1):

$$x_p = \pm \sqrt{k_{23}^2 + k_{12}^2 k_{23}/k_{14}}, \qquad (1)$$

where

$x_p = \frac{2(f_p - f_0)}{\Delta f_{3\,\mathrm{dB}}}$ normalized frequency of peak rejection

f_p = frequency of peak rejection, and

$\Delta f_{3\,\mathrm{dB}}$ = relative 3-dB bandwidth of general filter when $K_{14} = 0$.

Let

$$k_{12} = \frac{\Delta f_{12}}{\Delta f_{3\,\mathrm{dB}}} = \frac{25}{29} = 0.862$$

$$k_{23} = \frac{\Delta f_{23}}{\Delta f_{3\,\mathrm{dB}}} = \frac{15.5}{29} = 0.534$$

$$k_{14} = \frac{\Delta f_{14}}{\Delta f_{3\,\mathrm{dB}}} = \frac{3.3}{29} = 0.114.$$

Using (1), $x_p = \pm 1.94$. From Fig. 3, peak rejection has been obtained at frequencies of 2026 Mc/s and 1969.6 Mc/s. These actual frequencies correspond to normalized frequencies of $x_p = +1.79$ and $x_p = -2.10$. The average value of $|x_p|$ is 1.945. This checks quite closely with the theoretical value of $|x_p| = 1.94$. The asymmetrical locations of the actual frequencies of peak rejection can be primarily attributed to the frequency sensitivity of k_{14}.

The normalized valley frequencies can be determined from (2):

$$x_v = \pm \sqrt{2}\, x_p \qquad (2)$$

where

x_v = normalized valley frequency.

Using (2) and $x_p = \pm 1.94$, $x_v = \pm 2.75$. From Fig. 3, valleys have occurred at actual frequencies of 2041.5 Mc/s and 1959.5 Mc/s. These correspond to normalized frequencies of $x_v = +2.86$ and $x_v = -2.79$.

The peak rejection attainable can be determined from (3):

$$\text{I.L.} \cong 80 \log |x_p| + 20 \log \left[\frac{k_{12}^2 k_{23}}{2k_{14} d_2 |x_p|}\right] \text{dB} \qquad (3)$$

where

$d_2 = \frac{Q_T}{Q_{UL}}$ = normalized dissipation factor of second and third resonators

$Q_T = \frac{f_0}{\Delta f_{3\,\mathrm{dB}}}$ = total Q of filter, and

Q_{UL} = unloaded Q of second and third resonators.

Letting $\Delta f_{3\,\mathrm{dB}} = 29$ Mc/s and $f_0 = 2000$ Mc/s, $Q_T = 69$. For the levels of pass band dissipation losses encountered in this filter structure, Q_{UL} is estimated to be about 2300. Then $d_2 = 69/2300 = 0.03$. For $|x_p| = 1.94$, $k_{12} = 0.862$, $k_{23} = 0.534$, and $k_{14} = 0.114$, the theoretical peak rejection from (3) is 52.5 dB. This checks quite closely with measured values of peak rejection.

The insertion loss at the valley frequencies can be determined from (4):

$$\text{I.L.} \cong 80 \log |x_v| + 20 \log \left[\frac{k_{12}^2 k_{23}}{k_{14}(x_p^2 - x_v^2)}\right] \text{dB}. \qquad (4)$$

Letting $|x_v| = 1.75$ and using (4), a theoretical valley insertion loss of 40 dB is obtained. This can be compared to measured values of 39.2 dB and 38.9 dB.

Reasonably good correlation has been obtained between theoretical and measured performance. The general microwave filter structure discussed herein provides frequencies of peak rejection on both filter skirts without using techniques such as m-derived filter circuits or cascading band-pass and band-reject filters. The addition of the bridging coupling, K_{14}, can be implemented without modifying the overall filter form factor or substantially increasing the filter weight or fabrication cost.

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[4] M. Dishal, "Alignment and adjustment of synchronously-tuned multiple-resonant-circuit filters," *Proc. IRE*, vol. 39, pp. 1448–1455, November 1951.

Band-Pass Spurline Resonators

The spurline resonator, a term coined by Schiffman [1] to denote the particular type of transmission-line resonator shown in Fig. 1, is sometimes useful in the design and construction of transmission-line band-stop filters. Its open-wire-line equivalent (see Fig. 1) shows that it is naturally applicable to band-stop (or low-pass) transmission-line filters. A similar structure, which might be denoted a "band-pass spurline resonator" (to distinguish it from the band-stop type of spurline resonator), is shown in Fig. 2. Note that the main difference in the physical configuration of Fig. 2 from that of Fig. 1 is the grounding of the spurline. The main differences in the open-wire-line equivalent circuit is that the shunt stub is short-circuited at its end rather than open-circuited, and a transformer is required at Port 2. The band-pass spurline resonator may provide for alternative realizations of band-pass filters that have, either totally or in part, open-wire-line equivalent circuits of short-circuited shunt stubs alternating with commensurate length transmission lines. Examples are the parallel-coupled-resonator band-pass filter [3], [4] and the direct-coupled-stub band-pass filter [5], [6]. The transformer may, perhaps, be utilized to beneficially alter the impedance level within the filter. It is expected that the band-pass spurline resonator configuration will be most useful in filters of moderate bandwidth.

Design equations for spurline resonators are given in the following. Those for band-pass spurline resonators are believed to be new, while those for band-stop spurline resonators are an alternative form to equations originally derived by Schiffman for asymmetrical bandstop spurlines [7]. Define

$$A = \frac{Y_{oe}^a + Y_{oo}^a}{2} \qquad (1)$$

$$B = \frac{Y_{oe}^b + Y_{oo}^b}{2} \qquad (2)$$

$$D = \frac{Y_{oe}^a - Y_{oo}^a}{2} = \frac{Y_{oo}^b - Y_{oe}^b}{2}, \qquad (3)$$

where Y_{oo}^a, Y_{oe}^a are the odd- and even-mode admittances [2], respectively, of line a; and Y_{oo}^b, Y_{oe}^b are similarly defined for line b.

The inverse relationships for (1), (2), and (3) are

$$Y_{oe}^a = A - D \qquad (4)$$

$$Y_{oo}^a = A + D \qquad (5)$$

$$Y_{oe}^b = B - D \qquad (6)$$

$$Y_{oo}^b = B + D. \qquad (7)$$

Define also

$$k^2 = \frac{D^2}{AB}, \qquad (8)$$

where, by virtue of (1), (2), and (3),

$$0 \le k^2 \le 1. \qquad (9)$$

Manuscript received March 21, 1966.